Student Name: ______________________

Teacher: ___________________________

Sydney Technical High School

2024 | HIGHER SCHOOL CERTIFICATE **TRIAL EXAMINATION**

Mathematics Extension 1

General Instructions

- Reading Time – 10 minutes
- Working Time – 2 hours
- Write using black pen.
- Calculators approved by NESA may be used.
- A reference sheet is provided.
- For questions in Section II, show relevant mathematical reasoning and/or calculations.
- Marks may not be awarded for careless work or illegible writing.
- **Begin each question on a new page.**

Total marks: 70

Section I — 10 marks (pages 2-5)

- Attempt Questions 1 - 10
- Allow about 15 minutes for this section.

Section II — 60 marks (pages 6-14)

- Attempt Questions 11 - 14
- Allow about 1 hour and 45 minutes for this section.

Section I

10 marks
Attempt Questions 1–10
Allow about 15 minutes for this section.

Use the multiple-choice answer sheet for Questions 1–10.

---

1 The temperature of a cup of hot Milo milk ($T°C$) at time t seconds is modelled using Newton's Law of Cooling. If:

$$T(t) = 21 + 64e^{-0.016t}$$

what is the initial temperature of the cup of Milo?

A. $-0.016°C$

B. $64°C$

C. $21°C$

D. $85°C$

2 A group of 27 are voting for 4 people.

The person with the majority vote wins the vote and every person has only one vote.

What is the minimum number of votes a person can win with?

A. 6

B. 7

C. 8

D. 9

3 Which of the following is a first order linear differential equation?

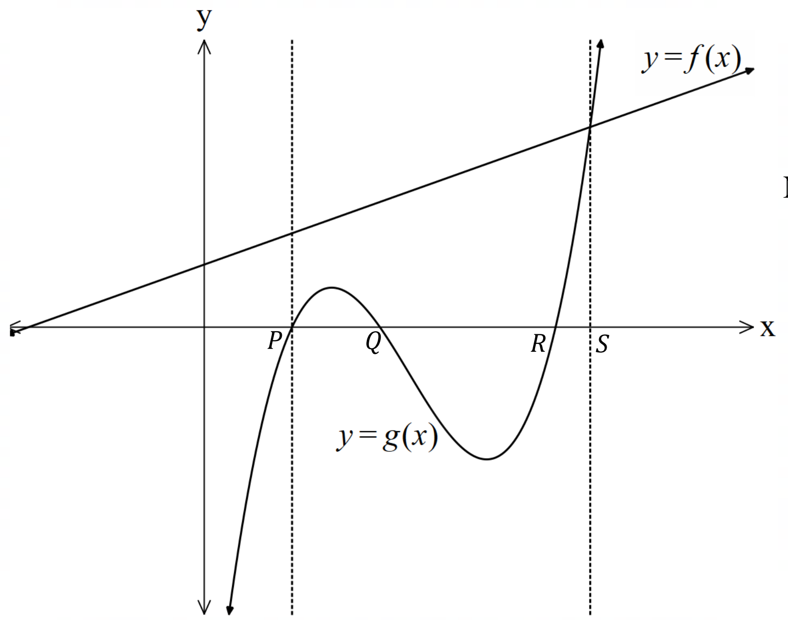
A. $y'' + y' \sin x = \cos x$

B. $\left(\frac{dy}{dx}\right) - 4y = x^6 e^x$

C. $\left(\frac{dy}{dx}\right)^2 + y \cos x = 5$

D. $yy' = y^2$

4 The diagram below shows the graphs $y = f(x)$ and $y = g(x)$.



Not to scale

It is known that:

$$\int_P^S f(x)dx = 18$$
$$\int_P^S g(x)dx = -5$$
$$\int_P^Q g(x)dx = 2$$
$$\int_Q^R g(x)dx = -9$$

What is the area between the curves $y = f(x)$ and $y = g(x)$ between $x = P$ and $x = S$?

A. $13\ u^2$

B. $14\ u^2$

C. $21\ u^2$

D. $23\ u^2$

5 Which of the following is the value of $\cos^{-1}(\cos\theta)$ given that $-\frac{\pi}{2} \leq \theta \leq 0$?

A. $\theta - \pi$

B. $\pi - \theta$

C. θ

D. $-\theta$

6 A group of 5 Jets and 3 Sharks are arranged in a circle. How many ways can this be done if Tony and Riff from the Jets must be next to each other?

A. $6! \times 2!$

B. $7! \times 2!$

C. $4! \times 3! \times 2!$

D. $4! \times 2! \times 2!$

7 The polynomial $P(x)$ has a remainder of 5 when divided by $(x+1)$ and $P(3) = 13$. The remainder when $P(x)$ is divided by $x^2 - 2x - 3$ would be:

A. $2x - 3$

B. $7x - 2$

C. $7x + 2$

D. $2x + 7$

8 Let $\underset{\sim}{a}$ and $\underset{\sim}{b}$ be two non-zero vectors and let the projection of $\underset{\sim}{a}$ onto $\underset{\sim}{b}$ be represented by the vector $\underset{\sim}{c}$.

Which of the following would be the projection of $10\underset{\sim}{a}$ onto $3\underset{\sim}{b}$?

A. $3\underset{\sim}{c}$

B. $3.\dot{3}\underset{\sim}{c}$

C. $10\underset{\sim}{c}$

D. $30\underset{\sim}{c}$

9 Which statement is always true for real numbers α and β where $-1 \leq \alpha < \beta \leq 1$?

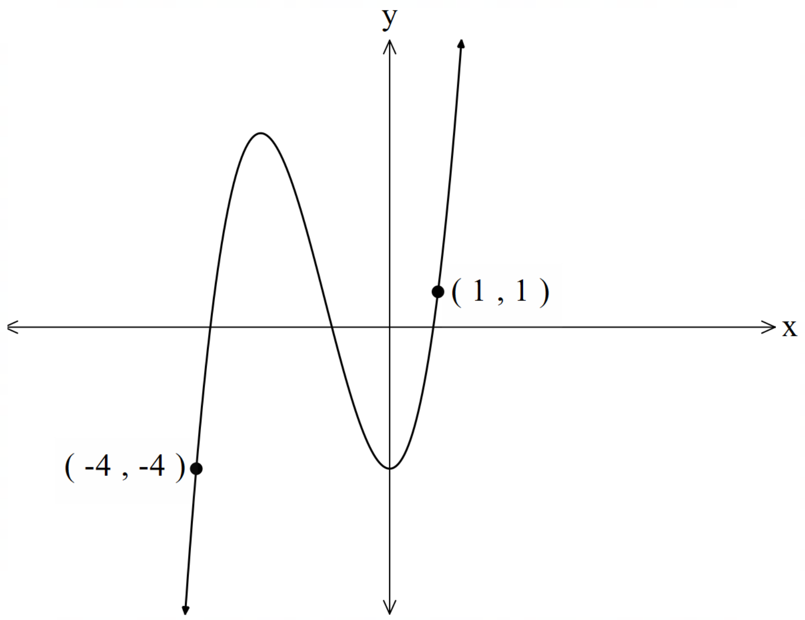
A. $\operatorname{cosec} \alpha < \operatorname{cosec} \beta$

B. $\sec \alpha < \sec \beta$

C. $\arccos \alpha < \arccos \beta$

D. $\sin^{-1} \alpha < \sin^{-1} \beta$

10 The graph of a function $y = f(x)$ is given below.



Which of the following functions will have an inverse relation whose graph could have more than 2 points that intersect with the line $y = x$?

A. $y = |f(x)|$

B. $y = \sqrt{f(x)}$

C. $y = f(|x|)$

D. $y = \frac{1}{f(x)}$

Section II

60 marks
Attempt Questions 11–14
Allow about 1 hour and 45 minutes for this section.

Answer each question on a new page in the answer booklet.

For questions in Section II, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (14 marks) Start a NEW page.

(a) Differentiate $6\tan^{-1}3x$. **2**

(b) If $\underset{\sim}{u} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$ and $\underset{\sim}{v} = \begin{pmatrix} x \\ -2 \end{pmatrix}$ are perpendicular, find the value of x. **2**

(c) Evaluate $\sin\left(2\cos^{-1}\frac{3}{5}\right)$ in exact form. **2**

(d) In how many ways can the letters in the word CAESAREAN be arranged in a line? **2**

(e) Given: $P(x) = x^3 + x^2 - 16x + 20$, find the value of:

i. $\alpha^2\beta\gamma + \alpha\beta^2\gamma + \alpha\beta\gamma^2$ **2**

ii. $\alpha^2 + \beta^2 + \gamma^2$ **2**

(f) Solve $y' = e^{3y}$, leaving x as a function of y. **2**

End of Question 11

Question 12 (16 marks) Start a NEW page.

(a) Solve $\cos\theta - \sqrt{3}\sin\theta = 1$ for $0 \le \theta \le 2\pi$ **3**

(b) Evaluate: **3**

$$\int_2^4 \frac{x+1}{\sqrt{x-2}}\,dx$$

using the substitution of $u = x - 2$.

(c) Use mathematical induction to prove that **3**

$$3^{2n} - 2^{2n} \text{ is divisible by } 5$$

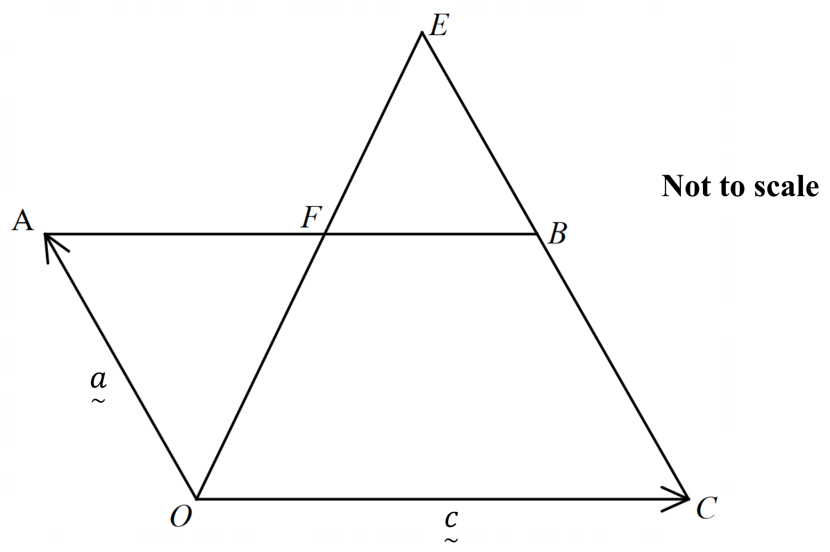
for all positive integers n.

Question 12 continues on page 8

Question 12 (continued)

(d) $OABC$ is a parallelogram where $\overrightarrow{OA} = \underset{\sim}{a}$ and $\overrightarrow{OC} = \underset{\sim}{c}$.

OF and CB are produced to meet at E such that $|\overrightarrow{BE}| = |\overrightarrow{OA}|$ as shown below:



i. Find an expression for $\overrightarrow{OE}$ in terms of $\underset{\sim}{a}$ and $\underset{\sim}{c}$. **1**

ii. Hence, show that F is the midpoint of $\overrightarrow{OE}$. **3**

(e) Solve the differential equation: **3**

$$\frac{dy}{dx} = 3xy$$

given $\frac{dy}{dx} = 12$ when $x = 2$.

End of Question 12

Question 13 (15 marks) Start a NEW page.

(a) Solve $\frac{3x}{x-1} \geq 2$ **3**

(b) Chicken pox is a highly contagious disease caused by the varicella-zoster virus. The probability of contracting chicken pox after exposure is 0.85 if the person is unvaccinated and drops to 0.18 if the person has had one dose of the vaccine. A group of 9 students are all exposed to the chicken pox in a room. It is known that in this group of students, 5 have been vaccinated against the chicken pox and 4 have not. All 9 have never had chicken pox previously.

i. Find an expression for the probability that exactly 3 of the 5 vaccinated students will still contract chicken pox. **1**

ii. Find the probability that exactly 3 of the 5 vaccinated students and all the unvaccinated students will contract chicken pox, to 2 decimal places. **1**

(c) Given that for all integers $1 \leq k + 1 \leq n$, **2**

$$^{n+1}C_{k+1} = {}^{n}C_{k} + {}^{n}C_{k+1}$$

find a possible set of values of n and r such that:

$$^{2\,025}C_{1\,940} + {}^{2\,024}C_{83} + {}^{2\,024}C_{84} = {}^{n}C_{r}$$

Question 13 continues on page 10

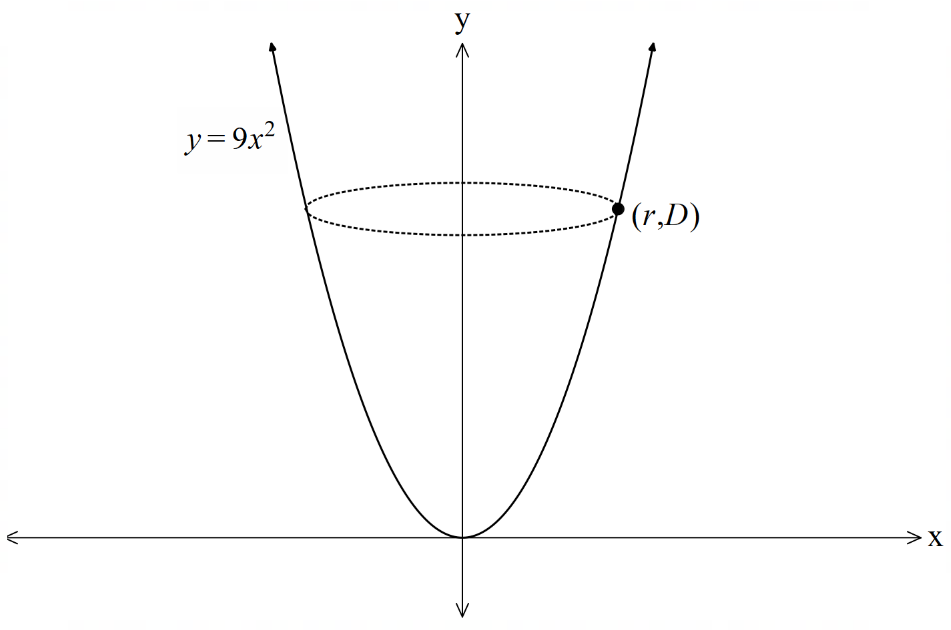
(d) A sample of 1200 Australian newborn babies were randomly selected.
Let $\hat{p}$ be the sample proportion of Australian newborn babies who had a head circumference at birth of 33.5 cm or less. Data from the World Health Organisation showed that 26% of newborn babies have a head circumference at birth of 33.5 cm or less.

i. Assuming the sample proportion is normally distributed, show that the standard deviation is 0.0127 to 3 significant figures. **1**

ii. Hence, using the standard normal distribution table and information on page 14, approximate the probability that of the 1200 Australian newborn babies, at least 300 of them have a head circumference at birth of 33.5 cm of less, giving your answer correct to 2 decimal places. **2**

Question 13 continues on page 11

(e) A pot plant self-watering system is created by rotating the parabola $y = 9x^2$ around the $y -$ axis between $y = 0$ and $y = D$.



i. Show that the volume of water at depth D can be given by: **2**

$$V = \frac{9}{2}\pi r^4$$

ii. The variable D is the depth of water left in the system as the water seeps into the soil after the plug at the vertex has been removed and r is the radius of the upper surface of the water at time t. All measurements are in centimeters and time is in days. **3**

Given $\frac{dr}{dt} = -0.5$, find the rate of change of the volume (V) when the radius is 4 cm.

End of Question 13

Question 14 (15 marks) Start a NEW page.

(a) Find: **3**

$$\int \sin 5x \cos 4x \, dx$$

(b) Consider the function:

$$f(x) = \arccos(2x - 1) - 2\arccos\sqrt{x} + 3 \text{ for } 0 \leq x \leq 1.$$

i. Show that $f'(x) = 0$ for $0 \leq x \leq 1$. **2**

ii. Hence, sketch the graph of $y = f(x)$. **2**

(c) Consider the monic polynomial:

$$P(x) = x^{m+1} - (m+1)x + m \text{ where } m \text{ is a positive integer.}$$

i. Show that $P(x)$ has a double root at $x = 1$. **2**

ii. By considering $P'(x)$, show that $P(x) \geq 0$ for all $x \geq 0$. **1**

(d)

i. In how many ways can n students be placed in two distinct rooms so that neither room is empty? Give your answer in simplest form. **1**

ii. In how many ways can five students be placed in three distinct rooms so that no room is empty? **2**

Question 14 continues on page 13

Question 14 (continued)

(e) The table below shows some values of a strictly monotonically increasing continuous function $f(x)$ and its derivative $f'(x)$. **2**

x	-1	0
$f(x)$	-6	-1
$f'(x)$	4	3

Find the gradient of the tangent at $x = -1$ of $f^{-1}(x)$.

End of paper

Sydney Technical High School

MULTIPLE CHOICE ANSWER SHEET

Completely fill the response oval representing the most correct answer.

Do not remove this sheet from the answer booklet.

1. A ○ B ○ C ○ D ●
2. A ○ B ○ C ● D ○
3. A ○ B ● C ○ D ○
4. A ○ B ○ C ○ D ●
5. A ○ B ○ C ○ D ●
6. A ● B ○ C ○ D ○
7. A ○ B ○ C ○ D ●
8. A ○ B ○ C ● D ○
9. A ○ B ○ C ○ D ●
10. A ○ B ○ C ○ D ●

Multiple Choice Answers

1. sub $t=0$

$T(0) = 21 + 64e^0$

$= 85°C$

2. $27 = 6 \times 4 + 3$

Person	1	2	3	4
Votes	6	6	7	8

↑ majority (person 4)

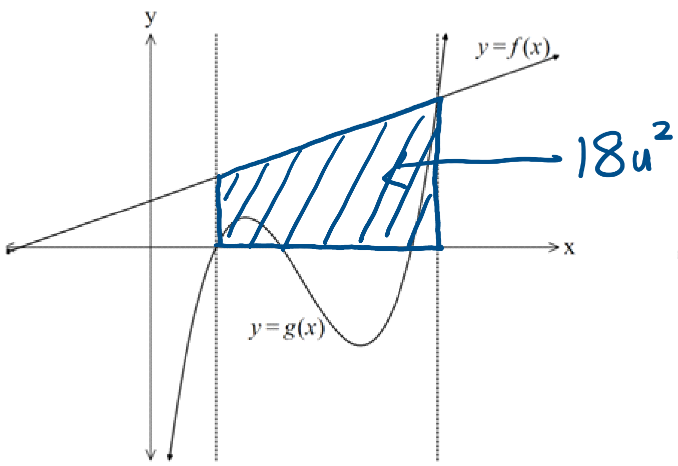
3. A. Second order

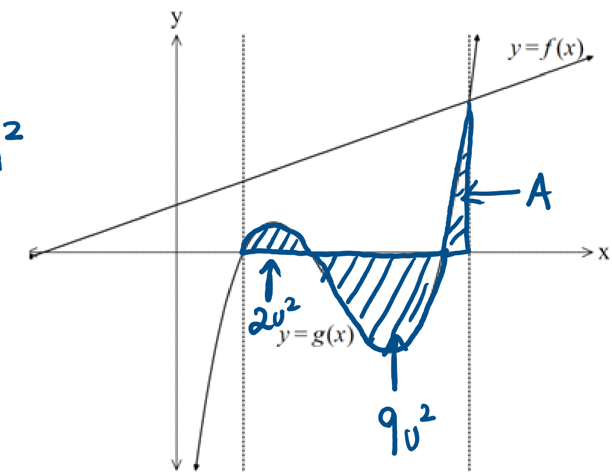
B. First order linear ←

C. Not linear

D. Not linear

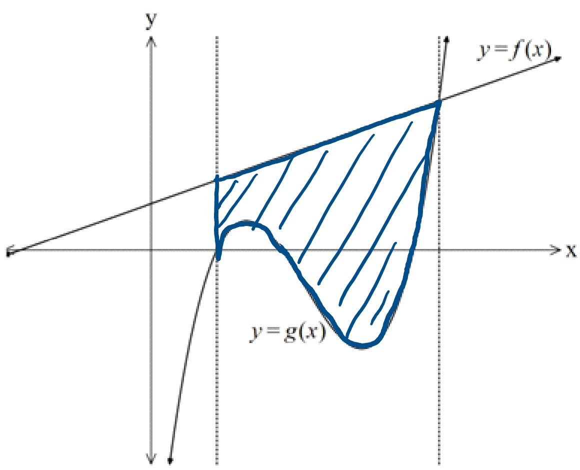
4.





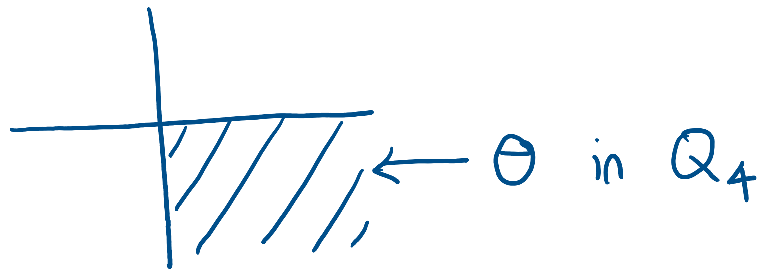
$2 + -9 + A = -5$

$A = 2$



$18 - 2 + 9 - 2 = 23u^2$

5.

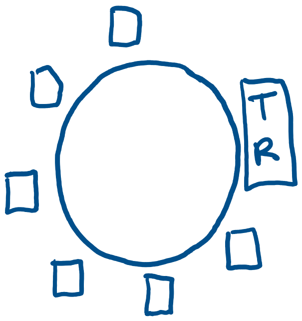


$\cos\theta = \cos(-\theta)$

$\cos^{-1}\cos\theta = \cos^{-1}\cos(-\theta)$

$= -\theta$

6.



$6! \times 2$

7. $P(x) = (x+1)(x-3)Q(x) + ax + b$

$P(-1) = 5$

$P(3) = 13$

A. $2(3) - 3 \neq 13$

B. $7(-1) - 2 \neq 5$

C. $7(3) + 2 \neq 13$

D. $2(-1) + 7 = 5$
$2(3) + 7 = 13$] Correct.

8. $\text{Proj}_{3\underset{\sim}{b}} 10\underset{\sim}{a} = \dfrac{10\underset{\sim}{a} \cdot 3\underset{\sim}{b}}{3\underset{\sim}{b} \cdot 3\underset{\sim}{b}} \times 3\underset{\sim}{b}$

$= \dfrac{30\underset{\sim}{a} \cdot \underset{\sim}{b}}{9\underset{\sim}{b} \cdot \underset{\sim}{b}} \times 3\underset{\sim}{b}$

$= \dfrac{30}{9} \times 3\underset{\sim}{c}$

$= 10\underset{\sim}{c}$

9. Test values:

A. $\operatorname{cosec}(-0.5) = -2.09$

$\operatorname{cosec}(-0.1) = -10.02$

B $\sec(-0.5) = 1.14$

$\sec(0) = 1$

C. $\arccos(-0.5) = 2.09$

$\arccos(-0.1) = 1.67$

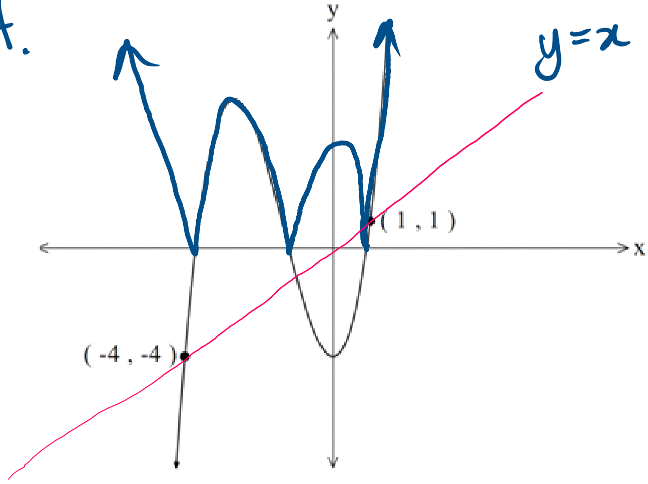
D. $\sin^{-1}(-0.5) = -0.52$

$\sin^{-1}(-0.1) = -0.10$

$\sin^{-1}(0.1) = 0.10$

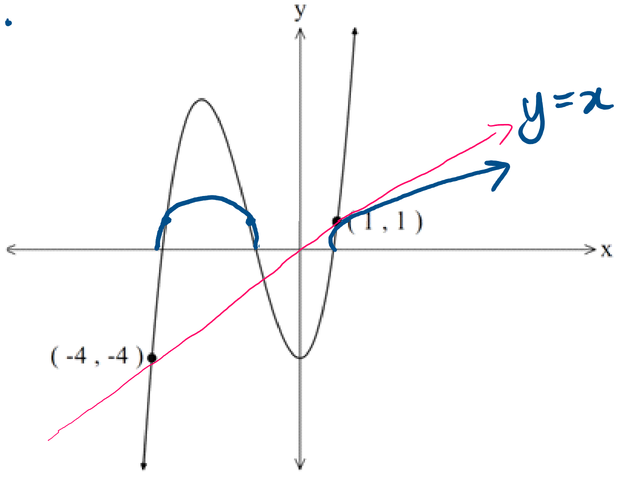
$\sin^{-1}(0.5) = 0.52$

10. A.



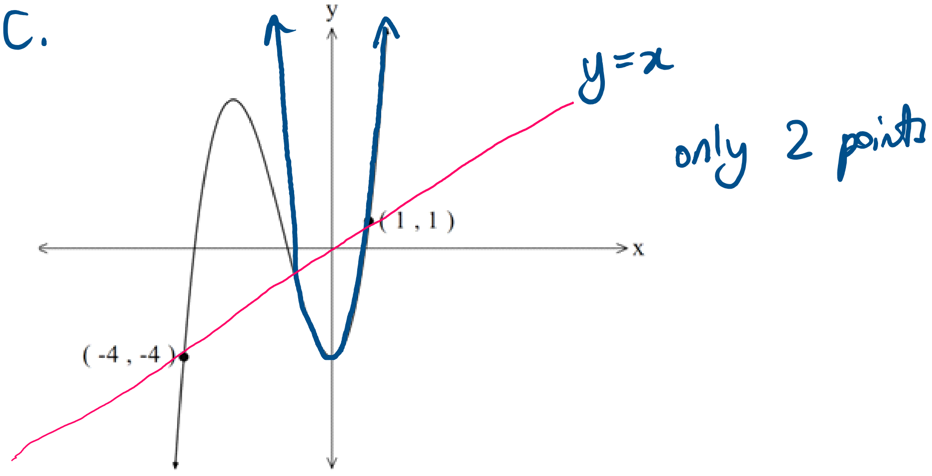
only 2 points

B.

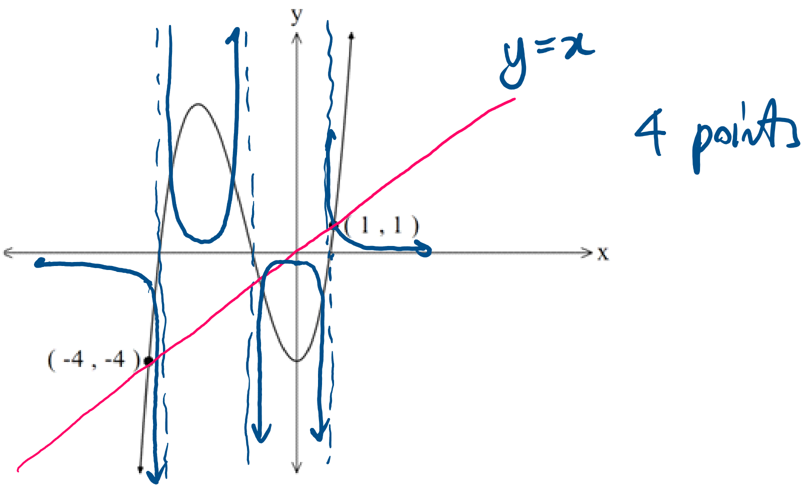


only 1 point

C.



D.



Question 11

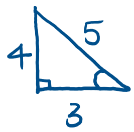
a) $6 \times \frac{3}{1+(3x)^2} = \frac{18}{1+9x^2}$

b) $\underset{\sim}{u} \cdot \underset{\sim}{v} = 0$

$3x - 2 = 0$

$x = \frac{2}{3}$

c) $2\sin\left(\cos^{-1}\frac{3}{5}\right)\cos\left(\cos^{-1}\frac{3}{5}\right)$



$= 2 \times \frac{4}{5} \times \frac{3}{5}$

$= \frac{24}{25}$

d) $\frac{9!}{2! \times 3!} = 30\ 240$

e) $\alpha + \beta + \gamma = -1$

$\alpha\beta + \alpha\gamma + \beta\gamma = -16$

$\alpha\beta\gamma = -20$

i. $\alpha^2\beta\gamma + \alpha\beta^2\gamma + \alpha\beta\gamma^2$

$= \alpha\beta\gamma(\alpha+\beta+\gamma)$

$= 20$

ii. $\alpha^2 + \beta^2 + \gamma^2 = (\alpha+\beta+\gamma)^2 - 2(\alpha\beta + \alpha\gamma + \beta\gamma)$

$= 1 - 2(-16)$

$= 33$

f) $y' = e^{3y}$

$\frac{dy}{dx} = e^{3y}$

$\frac{dx}{dy} = e^{-3y}$

$x = -\frac{1}{3}e^{-3y} + C$

Question 12

a) $\cos\theta - \sqrt{3}\sin\theta = 1 \quad 0 \leq \theta \leq 2\pi$

Using $R\cos(\theta + \alpha)$

$R\cos\alpha = 1$

$R\sin\alpha = \sqrt{3}$

$R = 2$

$\tan\alpha = \sqrt{3}$

$$\alpha = \frac{\pi}{3}$$

$$\therefore \; 2\cos\left(\theta + \frac{\pi}{3}\right) = 1$$

$$\cos\left(\theta + \frac{\pi}{3}\right) = \frac{1}{2}$$

$$\theta + \frac{\pi}{3} = \frac{\pi}{3}, 2\pi - \frac{\pi}{3}, 2\pi + \frac{\pi}{3}$$

$$\theta = 0, 2\pi - \frac{2\pi}{3}, 2\pi$$

$$= 0, \frac{4\pi}{3}, 2\pi$$

b) $\int_2^4 \frac{x+1}{\sqrt{x-2}}\,dx$

Let $u = x - 2 \longrightarrow x = u + 2$

$du = dx$

x	2	4
u	0	2

$$I = \int_0^2 \frac{u+2+1}{\sqrt{u}}\,du$$

$$= \int_0^2 u^{\frac{1}{2}} + 3u^{-\frac{1}{2}}\,du$$

$$= \left[\frac{u^{3/2}}{3/2} + \frac{3u^{\frac{1}{2}}}{\frac{1}{2}}\right]_0^2$$

$$= \frac{4}{3}\sqrt{2} + 6\sqrt{2} - 0$$

$$= \frac{22}{3}\sqrt{2}$$

c) Show true for $n=1$

$$3^{2(1)} - 2^{2(1)} = 9 - 4$$

$$= 5$$

$$= 5 \times 1$$

$\therefore$ True for $n=1$

Assume true for some $n=k$

$$3^{2k} - 2^{2k} = 5M, \quad M \in \mathbb{Z}^+$$

Prove true for some $n=k+1$

$$3^{2(k+1)} - 2^{2(k+1)} = 3^{2k} \times 3^2 - 2^{2k} \times 2^2$$

$$= 3^2(5M + 2^{2k}) - 2^{2k} \times 4 \quad \text{from assumption}$$

$= 9\times 5M + 9\times 2^{2k} - 4\times 2^{2k}$

$= 9\times 5M + 5\times 2^{2k}$

$= 5(9M + 2^{2k})$

since $9M + 2^{2k}$ is an integer

$\therefore 3^{2(k+1)} - 2^{2(k+1)}$ is divisible by 5 given $n=k$ is true.

$\therefore$ Hence by the principle of Mathematical Induction statement is true for $n \geq 1$.

d) i. $\vec{OE} = \vec{OC} + \vec{CB} + \vec{BE}$

Now $\vec{CB} = \underset{\sim}{a}$

$\vec{BE} = \underset{\sim}{a}$

$\vec{OE} = \underset{\sim}{c} + 2\underset{\sim}{a}$

ii. $\vec{OF} = \lambda \vec{OE}$

$= \lambda(\underset{\sim}{c} + 2\underset{\sim}{a})$

$\vec{OF} = \vec{OA} + \mu \vec{OB}$

$= \underset{\sim}{a} + \mu \underset{\sim}{c}$

$\therefore \lambda \underset{\sim}{c} + 2\lambda \underset{\sim}{a} = \underset{\sim}{a} + \mu \underset{\sim}{c}$

$\therefore 2\lambda = 1$

$\lambda = \frac{1}{2}$

$\therefore \mu = \lambda$

$= \frac{1}{2}$

$\therefore$ since $\vec{OF} = \frac{1}{2}\vec{OE}$

$\therefore$ F is midpoint of $\vec{OE}$

e) $\frac{dy}{dx} = 3xy$

$\int \frac{dy}{y} = \int 3x\,dx$

$\ln|y| = \frac{3}{2}x^2 + C$

when $\frac{dy}{dx} = 12$, $x=2$ so: $12 = 3\times 2\times y$

$y = 2$

$\therefore \ln 2 = \frac{3}{2}\times 2^2 + C$

$C = \ln 2 - 6$

$\therefore \ln|y| = \frac{3}{2}x^2 + \ln 2 - 6$

$y = e^{3/2 x^2 + \ln 2 - 6}$ __OR__ $y = 2e^{3/2 x^2 - 6}$

Question 13

a) $\frac{3x}{x-1} \geq 2 \quad , x \neq 1$

$3x(x-1) \geq 2(x-1)^2$

$0 \geq 2(x-1)^2 - 3x(x-1)$

$0 \geq (x-1)[2x - 2 - 3x]$

$0 \geq (x-1)(-2-x)$

$\therefore \quad x \leq -2 \quad \cup \quad x > 1$

b) i. ${}^5C_3 \times 0.18^3 \times 0.82^2$

ii. ${}^5C_3 \times 0.18^3 \times 0.82^2 \times 0.85^4 = 0.02$

c) ${}^{2025}C_{1940} + {}^{2024}C_{83} + {}^{2024}C_{84}$

$= {}^{2025}C_{1940} + {}^{2025}C_{84}$

$= {}^{2025}C_{85} + {}^{2025}C_{84}$

$= {}^{2026}C_{85}$

$\therefore n = 2026, \; r = 85$ OR $n = 2026, r = 1941$

d) i. $p = 0.26$

$\sigma = \sqrt{\frac{0.26 \times 0.74}{1200}}$

$= 0.0127$ (3 sig figs)

ii. $P(\hat{p} \geq \frac{300}{1200})$

$= P(\hat{p} \geq \frac{1}{4})$

$= P(z \geq \frac{0.25 - 0.26}{0.0127})$

$= P(z \geq -0.7874\ldots)$

$= 0.7852$ (from table)

$\doteqdot 0.79$ (2 d.p.)

e) i. $V = \pi\int_a^b x^2\,dy$

$= \pi\int_0^D \frac{y}{9}\,dy$

$= \pi\left[\frac{y^2}{18}\right]_0^D$

$= \frac{\pi}{18}D^2$

when $y=D$, $x=r$

$D = 9r^2$

$\therefore V = \frac{\pi}{18}(9r^2)^2$

$= \frac{9}{2}\pi r^4$

ii. $\frac{dr}{dt} = -0.5$

$\frac{dV}{dt} = \frac{dV}{dr} \times \frac{dr}{dt}$

$= \frac{dV}{dr} \times -0.5$

$V = \frac{9}{2}\pi r^4$

$\frac{dV}{dr} = \frac{9}{2}\pi \times 4r^3$

$= 18\pi r^3$

$\therefore \frac{dV}{dt} = 18\pi r^3 \times -0.5$

when $r=4$

$\frac{dV}{dt} = -576\pi$ cm^3/day

Question 14

a) $\int \sin 5x \cos 4x\,dx = \int \frac{1}{2}[\sin(5x-4x) + \sin(5x+4x)]\,dx$

$= \frac{1}{2}\int \sin x + \sin 9x\,dx$

$= \frac{1}{2}\left(-\cos x - \frac{1}{9}\cos 9x\right) + C$

$= -\frac{1}{2}\left(\cos x + \frac{\cos 9x}{9}\right) + C$

b) $f(x) = \arccos(2x-1) - 2\arccos\sqrt{x} + 3$

i. $f'(x) = \frac{-2}{\sqrt{1-(2x-1)^2}} + 2\,\frac{\frac{1}{2}x^{-\frac{1}{2}}}{\sqrt{1-x}}$

$= \frac{-2}{\sqrt{1-(4x^2-4x+1)}} + \frac{\frac{1}{\sqrt{x}}}{\sqrt{1-x}}$

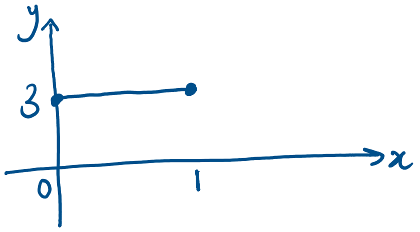
$$= \frac{-2}{\sqrt{4x-4x^2}} + \frac{1}{\sqrt{x}\sqrt{1-x}}$$

$$= \frac{-2}{2\sqrt{x-x^2}} + \frac{1}{\sqrt{x-x^2}}$$

$$= \frac{-1}{\sqrt{x-x^2}} + \frac{1}{\sqrt{x-x^2}}$$

$$= 0$$

ii.



c) i. $P(1) = 1^{m+1} - (m+1)\times 1 + m$

$= 1 - m - 1 + m$

$= 0$

$P'(x) = (m+1)x^m - (m+1)$

$P'(1) = (m+1)1^m - (m+1)$

$= (m+1) - (m+1)$

$= 0$

$\therefore$ Since $P(1) = P'(1) = 0$, $x=1$ is a double root for $P(x)$.

ii. $P'(x) = (m+1)x^m - (m+1)$

for $x > 1$

$(m+1)x^m > (m+1)$

$\therefore$ $(m+1)x^m - (m+1) > 0$

$P'(x) > 0$ → function is ALWAYS INCREASING.

for $0 < x < 1$

$(m+1)x^m < (m+1)$

$(m+1)x^m - (m+1) < 0$

$P'(x) < 0$ → function is DECREASING

from part i) $x=1$ is a double root.

i.e. minimum turning point on the x-axis (i.e. $P(x)=0$ is min)

$\therefore$ for $x \geq 0$, $P(x) \geq 0$.

$\therefore$ $P(x)$ starting at m and always increasing.

$\therefore$ $P(x) \geq 0$ for all $x \geq 0$.

d) i.

Rm 1		Rm 2
$^nC_1$	×	$^nC_{n-1}$
$^nC_2$	×	$^nC_{n-2}$

$^nC_3 \times {}^nC_{n-3}$

$\vdots$

$^nC_k \times {}^nC_{n-k}$

$\vdots$

$^nC_{n-2} \times {}^nC_2$

$^nC_{n-1} \times {}^nC_1$

when adding these you get:

$^nC_1 \times {}^nC_{n-1} + {}^nC_2 \times {}^nC_{n-2} + \dots + {}^nC_{n-2} \times {}^nC_2 + {}^nC_{n-1} \times {}^nC_1$

$= (1+1)^n - {}^nC_0 \times {}^nC_n - {}^nC_n \times {}^nC_0$

$= 2^n - 1 - 1$

$= 2^n - 2$

ii.

Rm 1	Rm 2	Rm 3
3	1	1
1	3	1
1	1	3
2	2	1
2	1	2
1	2	2

$\therefore\ {}^5C_3 \times {}^2C_1 \times {}^1C_1 \times 3 = 60$

${}^5C_2 \times {}^3C_2 \times {}^1C_1 \times 3 = 90$

Total number of ways : 150 ways.

e) $f^{-1}(-1) = 0$

since $f(0) = -1$

derivative of inverse:

$\frac{dy}{dx} f^{-1}(x)$

$= \frac{dx}{dy}\left(\frac{1}{f(y)}\right)$

$= \frac{1}{f'(y)}$

at $y=0$: $\frac{1}{f'(0)} = \frac{1}{3}$

$\therefore$ Gradient of tangent: $\frac{1}{3}$.